**Final Project**

**Problem 1**

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**Discussion of Results**

To analyze the performance of the three portfolios, I used CAPM regressions based on 2023 data to attribute both the realized return and realized risk over the holding period. The Return Attribution results help decompose each portfolio's return into two components: the portion explained by systematic risk (captured through exposure to SPY), and the idiosyncratic alpha not explained by the market. For instance, Portfolio C achieved a 28.12% realized return, with the return attribution showing 25.43% coming from market exposure and a positive alpha of 2.68%, indicating strong stock selection. In contrast, Portfolio A had a return of only 13.66%, with a high negative alpha of –11.63%, suggesting poor idiosyncratic performance despite similar beta exposure.

The Vol Attribution section quantifies the contribution of systematic and idiosyncratic volatility to overall portfolio volatility. Across all portfolios, we observe that volatility attribution is low (around 0.7% for each), indicating that the risk during the holding period was not a major driver of performance differences. Instead, alpha contributions played a much larger role, particularly in distinguishing the strong-performing Portfolio C from the underperforming Portfolios A and B. This supports the view that while all portfolios had similar beta exposure, differences in idiosyncratic return components (i.e., stock-picking skill) were the primary source of relative performance.

**Problem 2**

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Idiosyncratic Daily Standard Deviation (σ) for Each Stock:

Symbol ExpectedSigma RealizedSigma DeltaSigma

AAPL 0.008670 0.012066 0.003396

NVDA 0.025583 0.026030 0.000448

MSFT 0.012528 0.008318 -0.004210

AMZN 0.016498 0.012731 -0.003767

META 0.020389 0.019593 -0.000797

GOOGL 0.015381 0.014713 -0.000668

AVGO 0.016235 0.028379 0.012144

TSLA 0.027696 0.035570 0.007874

GOOG 0.015348 0.014531 -0.000817

BRK-B 0.006257 0.008251 0.001994

JPM 0.010918 0.013324 0.002406

LLY 0.017840 0.018057 0.000217

V 0.007532 0.009592 0.002060

XOM 0.014988 0.012256 -0.002732

UNH 0.013095 0.017127 0.004032

MA 0.008025 0.008812 0.000787

COST 0.010036 0.009822 -0.000214

PG 0.008857 0.009877 0.001020

WMT 0.009287 0.010868 0.001582

HD 0.010542 0.011505 0.000963

NFLX 0.021032 0.016307 -0.004725

JNJ 0.010031 0.009904 -0.000128

ABBV 0.012229 0.014750 0.002520

CRM 0.015695 0.019599 0.003904

BAC 0.013749 0.013075 -0.000673

ORCL 0.016886 0.017236 0.000350

MRK 0.011687 0.012964 0.001276

CVX 0.013675 0.011748 -0.001927

KO 0.007869 0.008413 0.000544

CSCO 0.011242 0.009984 -0.001257

WFC 0.014715 0.017019 0.002303

ACN 0.010470 0.014944 0.004474

NOW 0.015428 0.018038 0.002610

MCD 0.007760 0.010942 0.003182

PEP 0.008953 0.010606 0.001653

IBM 0.009057 0.013492 0.004436

DIS 0.013947 0.016362 0.002415

TMO 0.012268 0.011727 -0.000541

LIN 0.010693 0.008465 -0.002228

ABT 0.011544 0.011615 0.000071

AMD 0.024861 0.024571 -0.000291

ADBE 0.014661 0.021138 0.006477

PM 0.009284 0.013109 0.003825

ISRG 0.015189 0.014731 -0.000459

GE 0.013250 0.016652 0.003402

GS 0.012387 0.013420 0.001032

INTU 0.013319 0.014674 0.001355

CAT 0.015567 0.013228 -0.002339

QCOM 0.015541 0.018771 0.003230

TXN 0.011079 0.013499 0.002419

VZ 0.013920 0.013743 -0.000176

AXP 0.013092 0.013014 -0.000078

T 0.016565 0.013506 -0.003059

BKNG 0.013342 0.014240 0.000898

SPGI 0.009419 0.009636 0.000217

MS 0.012977 0.014116 0.001140

RTX 0.014361 0.011143 -0.003218

PLTR 0.037496 0.036167 -0.001329

PFE 0.013999 0.014470 0.000471

BLK 0.009435 0.009653 0.000218

DHR 0.013328 0.013084 -0.000244

NEE 0.016855 0.016190 -0.000665

HON 0.008831 0.010411 0.001579

CMCSA 0.012630 0.015070 0.002441

PGR 0.018643 0.012838 -0.005805

LOW 0.011936 0.012693 0.000757

AMGN 0.012333 0.015242 0.002909

UNP 0.013294 0.011363 -0.001930

TJX 0.008556 0.009589 0.001032

AMAT 0.016741 0.020519 0.003778

UBER 0.019644 0.023196 0.003552

C 0.013138 0.013927 0.000789

BSX 0.011828 0.009661 -0.002167

ETN 0.013911 0.012837 -0.001075

COP 0.017156 0.014427 -0.002729

BA 0.014845 0.021070 0.006225

BX 0.015916 0.014953 -0.000963

SYK 0.013055 0.011035 -0.002020

PANW 0.022137 0.025258 0.003121

ADP 0.011135 0.009607 -0.001527

FI 0.011296 0.009883 -0.001413

ANET 0.026446 0.019713 -0.006733

GILD 0.011949 0.015735 0.003785

BMY 0.011654 0.017979 0.006325

SCHW 0.024146 0.016266 -0.007880

TMUS 0.011461 0.010693 -0.000768

DE 0.015244 0.014138 -0.001107

ADI 0.012893 0.015440 0.002546

VRTX 0.014883 0.014579 -0.000303

SBUX 0.011951 0.022806 0.010855

MMC 0.008819 0.008956 0.000137

MDT 0.012348 0.010984 -0.001364

CB 0.012273 0.010777 -0.001496

LMT 0.011077 0.011007 -0.000070

KKR 0.014175 0.015178 0.001003

MU 0.019936 0.028228 0.008292

PLD 0.012994 0.014259 0.001265

LRCX 0.018365 0.021012 0.002647

EQIX 0.012450 0.014313 0.001863

Idiosyncratic Daily Risk Contribution (%) for Each Stock:

Symbol ExpectedPct RealizedPct DeltaPct

AAPL 47.57 72.46 24.89

NVDA 70.24 61.92 -8.32

MSFT 62.68 43.95 -18.73

AMZN 62.88 51.93 -10.95

META 66.04 73.01 6.97

GOOGL 64.52 69.64 5.12

AVGO 64.19 70.33 6.14

TSLA 69.74 77.41 7.67

GOOG 63.35 69.51 6.16

BRK-B 52.82 81.97 29.15

JPM 69.59 81.31 11.72

LLY 96.07 90.02 -6.06

V 58.87 81.63 22.76

XOM 91.40 103.91 12.51

UNH 97.00 99.99 2.99

MA 54.82 78.15 23.33

COST 69.41 69.03 -0.38

PG 88.70 110.18 21.48

WMT 89.01 95.50 6.49

HD 60.47 79.71 19.25

NFLX 78.91 77.44 -1.47

JNJ 92.56 110.24 17.68

ABBV 97.95 98.41 0.46

CRM 70.82 75.61 4.79

BAC 65.85 83.55 17.70

ORCL 78.94 72.35 -6.59

MRK 96.79 98.24 1.45

CVX 89.15 93.96 4.81

KO 86.22 110.31 24.09

CSCO 76.61 77.65 1.03

WFC 71.01 88.42 17.40

ACN 57.94 91.68 33.75

NOW 60.29 71.65 11.36

MCD 77.80 94.67 16.87

PEP 89.40 107.18 17.79

IBM 82.75 84.00 1.25

DIS 70.16 101.53 31.37

TMO 73.38 87.02 13.63

LIN 73.84 85.81 11.97

ABT 85.10 106.19 21.10

AMD 70.69 67.22 -3.47

ADBE 53.41 84.33 30.91

PM 79.90 109.01 29.12

ISRG 69.64 77.13 7.50

GE 74.93 75.46 0.53

GS 66.43 68.53 2.10

INTU 52.25 69.39 17.14

CAT 74.32 65.26 -9.06

QCOM 61.74 60.79 -0.95

TXN 53.68 61.55 7.87

VZ 92.57 108.54 15.97

AXP 63.46 73.40 9.94

T 94.65 113.52 18.87

BKNG 72.05 74.67 2.62

SPGI 46.11 92.73 46.62

MS 63.16 73.66 10.49

RTX 91.28 100.03 8.75

PLTR 73.59 79.89 6.30

PFE 93.23 99.05 5.82

BLK 45.74 73.56 27.82

DHR 75.99 87.22 11.24

NEE 89.07 103.14 14.07

HON 58.40 84.65 26.25

CMCSA 69.86 99.50 29.64

PGR 97.92 97.77 -0.15

LOW 64.51 83.95 19.44

AMGN 88.89 89.64 0.75

UNP 76.95 90.00 13.06

TJX 71.91 79.92 8.02

AMAT 61.48 59.44 -2.04

UBER 74.26 84.64 10.37

C 66.92 71.62 4.70

BSX 87.63 84.19 -3.44

ETN 69.65 52.35 -17.30

COP 90.87 103.11 12.24

BA 75.45 95.89 20.44

BX 52.93 68.30 15.37

SYK 78.15 87.72 9.57

PANW 84.11 85.23 1.12

ADP 70.97 100.25 29.28

FI 70.28 76.69 6.40

ANET 84.41 59.12 -25.29

GILD 87.58 102.88 15.30

BMY 91.34 101.25 9.91

SCHW 82.69 101.19 18.50

TMUS 93.21 96.49 3.28

DE 79.24 90.94 11.70

ADI 61.14 60.40 -0.74

VRTX 88.73 89.51 0.78

SBUX 70.20 94.53 24.33

MMC 69.70 105.43 35.74

MDT 83.73 99.65 15.92

CB 91.47 98.49 7.02

LMT 94.71 105.51 10.80

KKR 50.31 57.43 7.11

MU 76.18 75.41 -0.78

PLD 60.75 86.94 26.19

LRCX 64.25 59.33 -4.92

EQIX 66.56 85.11 18.55

**Discussion of Results for Part 2**

In Part 2, I constructed new portfolios optimized to maximize the Sharpe ratio under the CAPM framework, assuming zero alpha and using the average SPY return and risk-free rate prior to the holding period as expectations. Compared with the original portfolios, all three optimized portfolios (A, B, and C) demonstrated notable improvements in both total return and Sharpe ratio. For instance, Portfolio A’s return increased from **13.66% to 28.86%**, with a sharp reversal in alpha contribution from **–11.63% to +2.45%**, reflecting significant improvement in stock selection under the Sharpe-maximizing strategy. Similar enhancements were observed in Portfolios B and C, with their Sharpe ratios rising to **1.48 and 1.48**, respectively, outperforming their original versions. These results suggest that the optimized portfolios achieved a better balance between risk and return by increasing systematic exposure and reducing poorly performing idiosyncratic components.

When comparing expected versus realized idiosyncratic risk contributions, I observed notable deviations in several stocks. For example, **AAPL** and **V** showed higher realized idiosyncratic volatility than expected, with standard deviation differences of **+0.0034** and **+0.0021**, and risk contribution increases of **24.89% and 22.76%**, respectively. In contrast, names like **MSFT** and **AMZN** exhibited lower realized idiosyncratic risk than expected. These discrepancies highlight the limitations of ex-ante risk estimation and suggest that while CAPM provides a useful framework for constructing Sharpe-optimal portfolios, realized idiosyncratic risk remains partly unpredictable. Overall, I found that the optimization process led to portfolios with more favorable performance metrics and better-aligned risk-return profiles, supporting the utility of CAPM-based portfolio construction despite some deviations in risk realization.

**Problem 3:** Applications of the Normal Inverse Gaussian and Skew Normal Distributions in Finance

In the field of quantitative finance, accurately modeling asset returns is crucial for various applications, including risk management, pricing derivatives, portfolio construction, and credit analysis. Traditional models often assume that returns follow a normal distribution. However, real financial data frequently show patterns like fat tails (more extreme outcomes than expected under a normal distribution) and skewness (asymmetry), which contradict that assumption. To better reflect these characteristics, researchers and practitioners have turned to more flexible distributions—two popular ones being the Normal Inverse Gaussian (NIG) and the Skew Normal distributions.

**1. Normal Inverse Gaussian (NIG) in Finance**

The NIG distribution, part of the generalized hyperbolic family, was introduced to finance by Barndorff-Nielsen in 1997. It has four parameters that allow it to capture three important features of financial return distributions:

* Heavy tails, which represent higher chances of extreme outcomes
* Skewness, which captures asymmetry in returns
* Leptokurtosis, or high peaks, reflecting a sharper center than the normal distribution

These properties make the NIG distribution suitable for several financial applications:

* **Option pricing**: NIG is often used instead of Brownian motion to model the returns of the underlying asset. This helps produce more realistic implied volatility surfaces. Well-known models like CGMY and Variance Gamma are based on similar ideas.
* **Risk management**: For metrics like Value-at-Risk (VaR) and Expected Shortfall (ES), NIG offers better estimates of tail risk—especially under stress-testing conditions or for calculating regulatory capital.
* **High-frequency trading models**: NIG can describe the small-scale, rapid movements in prices more accurately than the normal distribution, making it useful for modeling limit order books and intraday volatility.
* **Interest rate modeling**: In fixed income markets, NIG is applied to term structure models where interest rates show jumps and fat tails.

Additionally, the NIG distribution is mathematically convenient. It has closed-form characteristic functions and is closed under convolution, which makes it useful both for theoretical work and simulations.

**2. Skew Normal Distribution in Finance**

The Skew Normal distribution is a natural extension of the regular normal distribution. By adding a skewness parameter, it can model asymmetry in return distributions while keeping many advantages of the Gaussian, such as simplicity and tractability. Although it doesn't account for heavy tails as well as NIG, it has other benefits:

* It’s relatively simple and easy to use
* It works well in situations where skewness is more important than kurtosis
* It integrates smoothly into models that already assume normality

In finance, the Skew Normal distribution is useful in several areas:

* **Modeling assets with a directional trend**: For example, stocks with consistent growth, IPOs, or commodities in a strong trend may show skewed return distributions.
* **Credit risk modeling**: In copula models for credit portfolios, Skew Normal helps capture asymmetric dependencies between defaults.
* **Performance evaluation**: When returns are skewed, standard ratios like the Sharpe ratio may be misleading. Metrics like the Sortino or Omega ratio benefit from using a distribution that reflects return asymmetry.
* **Bayesian modeling**: In Bayesian asset allocation or forecasting, the Skew Normal can serve as a prior when skewness is expected.

Compared to NIG, the Skew Normal is less ideal for modeling rare, extreme outcomes but works very well when we’re more interested in capturing moderate asymmetry—especially in diversified portfolios or individual stocks.

**Conclusion**

The Normal Inverse Gaussian and Skew Normal distributions are valuable tools in modern finance because they go beyond the limitations of the normal distribution. NIG is particularly strong for modeling extreme events, asymmetric risks, and sudden market jumps. The Skew Normal, while simpler, is effective in capturing directional biases in return data. By incorporating these distributions into financial models, analysts can better represent real-world return behavior, leading to more accurate risk assessments, improved forecasts, and better-informed financial decisions.

**Problem 4**

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Note: GC = Gaussian Copula with fitted marginals, MVN = Multivariate Normal

Best Fit Distribution Models and Parameters:

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Symbol Best Model Parameters

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SPY Normal (0.000985, 0.00823027)

AAPL GeneralizedT (7.32705184, 0.00176004, 0.01070133)

NVDA GeneralizedT (4.78942593, 0.0035959, 0.02167961)

MSFT GeneralizedT (7.77186949, 0.0016953, 0.01361571)

AMZN GeneralizedT (5.92185868, 0.00217428, 0.0168987)

META GeneralizedT (4.21960376, 0.00276747, 0.01569186)

GOOGL GeneralizedT (4.42004258, 0.0017383, 0.01418544)

AVGO GeneralizedT (4.30167917, 0.00138822, 0.01470093)

TSLA GeneralizedT (6.49117953, 0.00318664, 0.02780702)

GOOG GeneralizedT (4.59360274, 0.00182357, 0.01447821)

BRK-B GeneralizedT (6.74524696, 0.00067377, 0.00724056)

JPM GeneralizedT (3.51880692, 0.00159525, 0.00884377)

LLY GeneralizedT (3.23358817, 0.00165586, 0.01122477)

V GeneralizedT (9.51965663, 0.00095938, 0.00873213)

XOM GeneralizedT (7.88072967, -0.00021986, 0.01356079)

UNH GeneralizedT (3.37101843, 0.00022572, 0.0086698)

MA GeneralizedT (6.46819209, 0.00102464, 0.00888908)

COST GeneralizedT (4.62116261, 0.0015835, 0.009026)

PG GeneralizedT (5.51983912, 2.187e-05, 0.00757722)

WMT GeneralizedT (6.04506726, 0.00096688, 0.00742733)

HD GeneralizedT (4.53508785, 0.00070018, 0.01020933)

NFLX GeneralizedT (3.64484136, 0.00091162, 0.01564827)

JNJ GeneralizedT (3.60503826, -1.34e-06, 0.00700671)

ABBV GeneralizedT (4.01955609, 0.00037586, 0.00865251)

CRM GeneralizedT (5.10722255, 0.00200294, 0.01412393)

BAC GeneralizedT (4.26752079, -0.00011622, 0.01272531)

ORCL GeneralizedT (3.08185044, 0.0021289, 0.01082818)

MRK GeneralizedT (8.06842812, 0.00023836, 0.01032434)

CVX GeneralizedT (4.55344731, -8.92e-05, 0.01100463)

KO GeneralizedT (5.21548072, 5.878e-05, 0.00657088)

CSCO GeneralizedT (3.89493383, 0.00113959, 0.00847512)

WFC GeneralizedT (5.00366906, 0.00098326, 0.01369201)

ACN GeneralizedT (6.97220212, 0.00124094, 0.01151557)

NOW GeneralizedT (3.95042611, 0.00389179, 0.01473725)

MCD GeneralizedT (10.04277048, 0.00070566, 0.00788493)

PEP GeneralizedT (5.80499472, 0.00013102, 0.00761036)

IBM GeneralizedT (4.81519241, 0.00101536, 0.00758982)

DIS GeneralizedT (4.9084054, 0.00022155, 0.01281408)

TMO GeneralizedT (5.16094542, -9.467e-05, 0.01137947)

LIN GeneralizedT (3.17283373, 0.00135203, 0.00829867)

ABT GeneralizedT (6.22741603, -6.568e-05, 0.00995025)

AMD GeneralizedT (4.69751877, 0.00261995, 0.02264792)

ADBE GeneralizedT (5.878008, 0.00256884, 0.01635708)

PM GeneralizedT (8.1562215, 7.079e-05, 0.00900558)

ISRG GeneralizedT (4.70021557, 0.00154407, 0.0137455)

GE SkewNormal (1.82249234, -0.01199416, 0.02123692)

GS GeneralizedT (5.50564289, 0.00086959, 0.01220379)

INTU GeneralizedT (5.57011791, 0.00203526, 0.01485176)

CAT GeneralizedT (4.45013631, 0.00091466, 0.01320514)

QCOM GeneralizedT (5.220655, 0.00135796, 0.01561377)

TXN GeneralizedT (9.15095462, 0.00021763, 0.01336299)

VZ GeneralizedT (3.27124148, 0.00034792, 0.00910633)

AXP GeneralizedT (4.7185681, 0.00123521, 0.0123052)

T GeneralizedT (3.02149762, 6.288e-05, 0.01003867)

BKNG GeneralizedT (8.12043383, 0.00208597, 0.0135332)

SPGI GeneralizedT (4.16496256, 0.00173211, 0.00992714)

MS GeneralizedT (4.49436035, 0.00038078, 0.0122768)

RTX GeneralizedT (3.20868595, -0.00035334, 0.00909624)

PLTR GeneralizedT (3.11032753, 0.00134457, 0.02781572)

PFE GeneralizedT (4.09258281, -0.00192749, 0.01057259)

BLK GeneralizedT (7.93368605, 0.00051947, 0.012039)

DHR GeneralizedT (5.30548175, 0.00045194, 0.01192899)

NEE GeneralizedT (2.95117787, -0.00076447, 0.01067441)

HON GeneralizedT (5.72908595, 0.00037982, 0.00930099)

CMCSA GeneralizedT (4.55605759, 0.00090869, 0.01062443)

PGR GeneralizedT (2.64676743, 0.00124674, 0.00993869)

LOW NIG (63.63379063, 7.97831389, -0.00113827, 0.01408374)

AMGN GeneralizedT (5.94448029, 3.499e-05, 0.01074017)

UNP GeneralizedT (3.98526301, 0.0003117, 0.0099356)

TJX GeneralizedT (10.19180221, 0.00080818, 0.00902108)

AMAT SkewNormal (1.40541978, -0.01582076, 0.02799633)

UBER GeneralizedT (9.44055342, 0.00354852, 0.0200986)

C GeneralizedT (4.16803442, 0.00073837, 0.01184787)

BSX GeneralizedT (3.54086, 0.00104552, 0.00865211)

ETN GeneralizedT (3.8783439, 0.00236338, 0.01202696)

COP GeneralizedT (5.8301282, 0.00019623, 0.01449162)

BA GeneralizedT (4.70300787, 0.0013277, 0.01309473)

BX GeneralizedT (6.3059333, 0.00277636, 0.01820997)

SYK GeneralizedT (2.66310918, 0.00096561, 0.008678)

PANW GeneralizedT (3.32214521, 0.0032621, 0.01555676)

ADP GeneralizedT (3.39026684, 0.00057481, 0.00851731)

FI GeneralizedT (3.72162785, 0.00093909, 0.00885607)

ANET GeneralizedT (2.74406401, 0.00238797, 0.01557043)

GILD GeneralizedT (8.56930944, 4.86e-05, 0.01120598)

BMY GeneralizedT (4.35979899, -0.00095604, 0.00906315)

SCHW GeneralizedT (2.83913538, -0.0001954, 0.01585662)

TMUS GeneralizedT (5.56930757, 0.00113596, 0.00953262)

DE GeneralizedT (5.6007237, 0.00034151, 0.01369609)

ADI GeneralizedT (6.36393808, 0.0009286, 0.0135401)

VRTX GeneralizedT (4.00697129, 0.00137905, 0.01041982)

SBUX GeneralizedT (4.18938608, -8.21e-06, 0.00961351)

MMC GeneralizedT (5.36067464, 0.00112148, 0.00833078)

MDT GeneralizedT (4.58297189, 0.00045716, 0.01037367)

CB GeneralizedT (5.69179653, 0.0003543, 0.01032532)

LMT GeneralizedT (3.70332478, -0.00016025, 0.00739249)

KKR GeneralizedT (7.22819361, 0.00254548, 0.01701882)

MU SkewNormal (2.21502803, -0.02111968, 0.03275535)

PLD GeneralizedT (6.67570246, 0.00088595, 0.01394534)

LRCX GeneralizedT (4.95510526, 0.00173803, 0.01803918)

EQIX GeneralizedT (5.29593928, 0.00116643, 0.01221178)

**Discussion of Results for Part 4**

In Part 4, I implemented a distribution-fitting process for each stock using pre-holding period data, considering the Normal, Generalized T, Normal Inverse Gaussian (NIG), and Skew Normal distributions. Based on goodness-of-fit criteria, the **Generalized T distribution was selected as the best fit for the majority of the stocks**, while a few stocks such as **GE, AMAT, and MU** were better captured by the **Skew Normal distribution**, and **LOW** was best fitted by **NIG**. For this analysis, I assumed a zero expected return for each stock, and used the fitted marginals to construct portfolio risk using two methods: a **Gaussian Copula (GC)** approach and a **Multivariate Normal (MVN)** approach.

When comparing the two methods, I found that **VaR and ES under GC were consistently lower than those under MVN for most portfolios**, indicating that the copula model may better capture tail dependence and joint behavior among the fitted non-normal distributions. For example, for Portfolio C, the 1-day 95% **Expected Shortfall (ES)** was **0.02132 under GC**, compared to **0.02009 under MVN**, suggesting slightly heavier tails in the GC-driven joint distribution due to the fitted marginals. For the combined portfolio**, ES under GC was 0.01735**, slightly higher than **0.01657 under MVN,** again showing that modeling non-Gaussian behavior using fitted marginals and copulas may better reflect real downside risk. This difference illustrates the importance of capturing marginal distribution features like skewness, excess kurtosis, and tail fatness when estimating portfolio risk.

Overall, I conclude that while the Multivariate Normal assumption provides a simpler modeling framework, the Gaussian Copula with individually fitted marginals produces a more flexible and realistic risk model—especially in the presence of asymmetry and heavy tails in asset returns.

**Problem 5:** Risk Parity Portfolios Based on Expected Shortfall

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Portfolio A - Risk Contributions:

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Symbol Risk Parity Weight Marginal Contribution % of Total Risk

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WFC 1.92 % 1.5595 3.00 %

ETN 2.70 % 1.1747 3.18 %

AMZN 2.49 % 1.2468 3.11 %

QCOM 1.77 % 1.7258 3.05 %

LMT 5.17 % 0.6021 3.11 %

KO 5.44 % 0.5465 2.97 %

JNJ 5.07 % 0.6100 3.09 %

ISRG 2.36 % 1.3585 3.20 %

XOM 3.14 % 0.9323 2.92 %

MDT 3.22 % 0.9202 2.96 %

DHR 2.65 % 1.0983 2.91 %

PLD 2.12 % 1.4254 3.02 %

BA 2.67 % 1.1442 3.06 %

PG 6.19 % 0.4827 2.99 %

MRK 6.64 % 0.4575 3.04 %

AMD 1.84 % 1.6426 3.02 %

BX 1.44 % 1.9845 2.86 %

PM 3.77 % 0.8127 3.07 %

SCHW 1.46 % 2.0175 2.94 %

VZ 4.50 % 0.6858 3.09 %

COP 3.00 % 1.0085 3.02 %

ADI 2.23 % 1.3594 3.03 %

BAC 1.91 % 1.5687 3.00 %

NOW 2.24 % 1.3659 3.06 %

TMO 2.64 % 1.1764 3.10 %

CVX 3.13 % 0.9607 3.00 %

ANET 2.07 % 1.4695 3.03 %

NVDA 2.17 % 1.3797 2.99 %

GE 3.56 % 0.9102 3.24 %

GILD 3.75 % 0.8027 3.01 %

MU 2.38 % 1.2535 2.99 %

CMCSA 2.43 % 1.2242 2.98 %

DIS 1.92 % 1.5276 2.94 %

Target risk contribution per asset: 3.03%

Maximum deviation from target: 0.21%

Portfolio B - Risk Contributions:

----------------------------------------------------------------------

Symbol Risk Parity Weight Marginal Contribution % of Total Risk

----------------------------------------------------------------------

AXP 2.08 % 1.4485 3.02 %

HON 2.58 % 1.1664 3.01 %

META 2.46 % 1.2431 3.06 %

NFLX 2.06 % 1.4857 3.06 %

PGR 3.55 % 0.8736 3.10 %

LLY 4.77 % 0.6292 3.00 %

JPM 2.83 % 1.0816 3.07 %

VRTX 3.89 % 0.7725 3.00 %

TJX 3.94 % 0.7481 2.95 %

EQIX 2.40 % 1.2352 2.96 %

AAPL 2.82 % 1.0623 3.00 %

FI 2.73 % 1.1002 3.00 %

DE 2.42 % 1.2281 2.98 %

SBUX 2.58 % 1.1595 2.99 %

GOOGL 2.56 % 1.2307 3.15 %

T 3.59 % 0.8529 3.06 %

ABT 4.00 % 0.7731 3.09 %

BMY 4.02 % 0.7633 3.07 %

MS 2.24 % 1.3319 2.98 %

CRM 2.84 % 1.0785 3.07 %

PFE 3.37 % 0.9047 3.05 %

SPGI 1.97 % 1.5445 3.05 %

BRK-B 3.39 % 0.8978 3.04 %

ADBE 1.80 % 1.6619 2.99 %

ACN 2.65 % 1.1344 3.00 %

AMGN 3.87 % 0.7775 3.01 %

LIN 3.22 % 0.9334 3.00 %

V 3.51 % 0.8592 3.02 %

WMT 4.58 % 0.6415 2.94 %

AMAT 2.47 % 1.2326 3.04 %

CAT 2.27 % 1.3206 2.99 %

RTX 3.05 % 1.0100 3.08 %

UNP 3.47 % 0.9069 3.15 %

Target risk contribution per asset: 3.03%

Maximum deviation from target: 0.12%

Portfolio C - Risk Contributions:

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Symbol Risk Parity Weight Marginal Contribution % of Total Risk

----------------------------------------------------------------------

IBM 4.64 % 0.6801 3.15 %

TXN 2.24 % 1.3640 3.06 %

ADP 2.54 % 1.2464 3.16 %

GOOG 2.34 % 1.2992 3.04 %

ORCL 2.06 % 1.4514 3.00 %

BSX 3.80 % 0.7969 3.03 %

UNH 5.67 % 0.5447 3.09 %

TMUS 4.85 % 0.6447 3.13 %

SYK 2.92 % 1.0243 2.99 %

GS 2.47 % 1.2140 2.99 %

UBER 2.10 % 1.4330 3.01 %

AVGO 2.31 % 1.2869 2.97 %

MMC 3.18 % 0.9671 3.07 %

CSCO 2.53 % 1.2025 3.04 %

PLTR 1.20 % 2.4976 2.99 %

MA 2.96 % 1.0121 3.00 %

C 2.44 % 1.2387 3.02 %

BKNG 2.94 % 0.9905 2.92 %

MCD 4.82 % 0.6311 3.04 %

LOW 2.69 % 1.1622 3.13 %

HD 2.71 % 1.1648 3.15 %

INTU 1.72 % 1.7158 2.96 %

LRCX 2.02 % 1.4808 2.98 %

KKR 1.74 % 1.6626 2.90 %

COST 3.95 % 0.7909 3.13 %

NEE 2.28 % 1.3169 3.01 %

ABBV 5.65 % 0.5138 2.90 %

TSLA 1.69 % 1.7287 2.93 %

MSFT 3.06 % 0.9669 2.96 %

PEP 5.32 % 0.5805 3.09 %

CB 4.39 % 0.6920 3.04 %

PANW 2.21 % 1.3721 3.04 %

BLK 2.54 % 1.2102 3.08 %

Target risk contribution per asset: 3.03%

Maximum deviation from target: 0.13%

Risk Parity Portfolio Attribution (for holding period):

A screenshot of a computer screen

AI-generated content may be incorrect.

=== Comparison of All Three Portfolio Strategies ===

A screenshot of a computer screen

AI-generated content may be incorrect.A screen shot of a computer

AI-generated content may be incorrect.

**Discussion Paragraph for Part 5**

In Part 5, I constructed **risk parity portfolios** for each sub-portfolio using **Expected Shortfall (ES)** as the risk metric and the best-fit marginal distributions from Part 4. The objective was to equalize the **tail risk contribution** of each asset to the overall portfolio ES, providing a more balanced risk allocation compared to the original or Sharpe-optimal portfolios. I then reran the CAPM-based attribution using the previously estimated beta values to analyze return and volatility contributions.

Compared to the original portfolios in Part 1, all three **risk parity portfolios achieved lower 1-day 95% ES and VaR**, confirming a more diversified and stable risk profile. Notably, **Portfolio C’s ES dropped by 27.81%, Portfolio A’s by 17.72%,** and **Portfolio B’s by 11.03%.** This demonstrates that the risk parity approach effectively reduced downside risk across the board. Similarly, VaR decreased significantly, especially **for Portfolio C with a 20.20% reduction.**

From a return perspective, the risk parity strategy was particularly successful in **Portfolio C,** which achieved the **highest overall return of 30.61%**—outperforming both the original and the Sharpe-optimal portfolios. Furthermore, it also generated the **highest idiosyncratic return of 8.22%,** suggesting that the tail-risk balancing process may have indirectly enhanced exposure to alpha-generating assets.

In addition to strong returns, the **risk parity portfolios also had the lowest market exposure**, with the **combined portfolio beta dropping to just 0.86**, compared to 0.95 in the original and 1.01 in the Sharpe-optimized version. This indicates that risk parity portfolios achieved improved outcomes not by increasing risk, but by allocating more efficiently across less correlated, lower-beta assets.

Overall, I found that **risk parity portfolios offered a compelling trade-off between risk and return**, especially in managing downside exposure while still preserving the opportunity for strong performance. Portfolio C, in particular, stood out as the most successful case, combining the best return, the strongest alpha contribution, and the most significant risk reduction.